Int. J. Heat Mass Transfer. Vol. 21, pp. 735–743
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0017-9310/78/0601-0735 $02.00/0

COMPUTER MODEL OF THERMAL REGENERATORS WITH VARIABLE MASS FLOW RATES

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(*Received* 30 *June* 1977 *and in revised form* 30 *October* 1977)

Abstract—Regenerative heat exchangers which require constant exit temperature of the cold fluid, are equipped with a control mechanism which allows either a certain quantity of the incoming fluid to by-pass the regenerator and mix directly with the outgoing fluid-stream or regulates the mixing of two exit fluid-streams from two regenerators which are out of phase by 1/2 of the cold period, in order to maintain the constant temperature. In either case, the variable cold fluid flow-rate is controlled by its exit temperature. This paper describes the solution of the differential equations which govern this non-linear heat-transfer process. The temperature dependence of the solid and fluid properties and the heat-transfer coefficient is taken into account. This method can be utilized to aid the design of new regenerators or to improve the performance of existing installations. The solution is general and is applicable to any type of regenerator.

NOMENCLATURE

a_0, a_1, a_2, numerical constants;
A, heat transfer area [m²];
b_1, b_2, b_3, numerical constants;
B_1, B_2, B_3, numerical constants defined by the equation (1.10);
C, volumetric specific heat of solid [J/m³ K];
c, volumetric specific heat of fluid [J/scm K];
C_0, C_1, C_2, numerical constant defined by the equation (2.11);
$f(t)$, fluid inlet temperature [°C];
F_c, temperature correction factor;
F_R, roughness correction factor;
$F(\lambda)$, function defined by the equation (22);
$F_1(\lambda)$, function defined by the equation (23);
h, heat-transfer coefficient [W/m² K];
h_c, convective heat-transfer coefficient [W/m² K];
h_r, radiative heat-transfer coefficient [W/m² K];
h^*, corrected heat-transfer coefficient [W/m² K];
$\hbar$, volumetric specific enthalpy [J/scm];
j, jth step;
J, total time steps;
k, thermal conductivity of the solid [W/m K];
K_1, K_2, K_3, numerical constants defined by the equations (2.9);
L, height of the regenerator [m];
$\dot{m}$, fluid flow-rate [scms];
n, the nth node;
N, total number of nodes;
Q_0, heat transferred/unit time, defined in the text [W];
Q_p, heat transferred per period [J/cycle];
r, radius [m];
t, time [s];
t_0, fluid period [s];
t_r, reversal time [s];
T, solid temperature [°C];
U, dimensionless fluid temperature;
V, dimensionless solid temperature;
V_s, volume of the solid [m³];
z, distance from the regenerator entrance [m].

Greek symbols

β, ratio of radii;
δ, solid semi-thickness [m];
ζ, dimensionless time;
η_r, thermodynamic efficiency of the regenerator;
η_t, thermodynamic efficiency;
Θ, fluid temperature [°C];
λ, ratio of fluid flow-rate/maximum flow-rate;
Λ, dimensionless length of the regenerator;
μ, ratio of periods;
ξ, dimensionless distance;
Π, dimensionless period;
τ, Fourier number;
τ_m, mean harmonic Fourier number;
Φ, correction factor.

Subscripts

b, blast;
e, exit;
i, inlet;
max, maximum;
n, quantities at the nth node;
1,2, refer to the quantities of the hot and cold stove respectively.

Superscripts

j, quantities at the jth time step;

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quantities at the cold period;
space average;
temperature average;
time average.

1. INTRODUCTION

MANY industrial processes require the transfer of heat of a hot fluid to a relatively colder one. This is usually accomplished by indirect heat transfer (conventional heat exchanger) which is characterized by separation of two fluids by a thin thermally conducting wall (usually metal), across which the hotter fluid continuously transfers its heat to the colder one. In the case where the fluids are high temperature corrosive gases, the capital expenditure required for efficient indirect equipment escalates rapidly because (i) special materials must be used in its construction and (ii) the physical dimensions of the units must be dramatically increased to compensate for the poor heat-transfer characteristics of the gases. A partial solution to the cost and corrosion problems is achieved by the use of regenerators where the transfer of heat from one fluid to the other is accomplished by the alternate heating and cooling of a corrosion resistant refractory solid, with embedded channels, through which the hot and cold gases alternately flow. If t'_0 and t_0 are respectively the duration of time that the cold and hot gas flows through the regenerator, then for a continuous process the cold–hot gas cycle will be repeated with a frequency $1/(t_0+t'_0+t_r)$.

Here t_r is the so-called reversal period when there is no gas flow. After some time, the regenerator temperature will reach a state called cyclic equilibrium. At this point, the temperature throughout the regenerator and of the gases will be cyclic functions of time.

Numerous articles have been presented on the subject the last four decades starting with the pioneer work of Angelius [1], Shumann [2], Ackermann [3] and the outstanding analysis of Hausen [4, 5, 6].

In this paper we investigate the blast furnace stove regenerators such as those used in the steel industry. (Our analysis is also applicable to any other type.) Here, a stream of hot air (blast air) of a constant temperature (the blast temperature) is required. Thus, the exit temperature of the air from the stove† should be constant. Since the air exit temperature from a stove decreases monotonically with time, an arrangement must be made to yield the required constant temperature. Here we investigate two such arrangements commonly used in practice. The first is called "series parallel" where one stove is on air and one or more on gas (combustion gas and/or hot waste gas being the hot fluid). The second is called "staggered" and in this case, two stoves are on air and two on gas. In the series arrangement, some of the incoming air bypasses the stove and is mixed with exit air stream to produce the required blast temperature, while in the "staggered" system, the constant blast temperature is obtained by mixing the exit air of the two stoves which are maintained out of phase by one half of a period ($t'_0/2$). In this application, the blast temperature is equal to the final air exit temperature in the "series" arrangement, while in the "staggered" arrangement, it is equal to the air exit temperature in the middle of the air period. It is clear, that in either case, the air flow-rate through the stove must be controlled by the air exit temperature. Thus, the entire heat-transfer process, by virtue of the flow-rate dependent heat-transfer coefficients, becomes a functional of the air exit temperature.

There is an important difference with respect to the variation of the air flow-rate through the stoves in the two arrangements. In the "series" arrangement the air flow-rate varies, in extreme cases, between approximately 75 and 100% of the blast flow-rate, while in the "staggered" this variation is between 0 and 100%. All previous authors have considered only constant flow-rates. If the problem required a variable flow-rate as in the case here, an average flow rate would have to be used [7]. Recently, Hausen [8] has shown how to correct for the variable flow-rate if the variation is within ±25% from its average value with a maximum error in the heat-transfer coefficient of ±7%. This correction covers the practical cases for the "series" arrangement. However, in the "staggered" arrangement, the average flow-rate is *a priori* known and is equal to one-half of the blast flow. Moreover, the variation of the flow-rate through the stove is ±100% from its average value and the previous work therefore does not apply. Willmott [9] has also obtained a solution to the "series" stove arrangement, by assuming a linear variation of the mass flow-rate of air.

In the present analysis the solution of the stove problem with variable flow-rate, which is controlled by the air exit temperature, is obtained by simulation on a digital computer.

2. GOVERNING EQUATIONS AND NORMALIZATION

The differential equations describing the thermal behavior of the regenerator, under the restrictions mentioned below, are:

$$h^*A(\Theta - T) = CV_s\partial T/\partial t \quad (1)$$

$$h^*A(T-\Theta) = \dot{m}cL\partial\theta/\partial z. \quad (2)$$

The restrictions are: (i) The thermal behavior of all channels is identical. (ii) Fouling is neglected. (iii) The gas composition remains constant. (iv) The heat capacity of the gas in the channels is small relative to the heat capacity of the solid. (v) The effect of the thermal conduction in the direction of flow can be neglected.† (vi) The effects of the residual gas during the reversal period are neglected. (vii) The solid temperature in the direction normal to the flow, is lumped, and the effect of its thermal resistance has been incorporated in the corrected heat-transfer coefficient h^* defined by Hausen [5] as,

$$1/h^* = (1/h)(1+\Phi h\delta/3k) \quad (3)$$

† Henceforth for simplicity the term "stove" will be used to indicate the blast furnace stove regenerator.

† A justification of this restriction is given in Appendix 3.

where Φ is a correction factor, which is determined by solving a simpler cyclic equilibrium problem that of constant heat flux into the solid [5]. This correction factor is a function of the three variables,

$2/\tau_m = 1/\tau + 1/\tau'$ harmonic mean Fourier number (4)

$\beta = r_2/r_1$ ratio of outside to the inside radii of a cylindrical channel (5)

$\mu = t_0/t'_0$ ratio of the periods. (6)

The equations for obtaining Φ are given in [10] and they are incorporated in the program.

Equations (1) and (2) must be solved subject to equilibrium conditions

$$T(0, z) = T'(t'_0, L-z') \quad (7a)$$

$$T(t_0, z) = T'(0, L-z') \quad (7b)$$

for $0 \leqslant z, z' \leqslant L$ where z, z' are measured always from the entrance of the hot and cold gas respectively and the boundary conditions,

$$\Theta_i = f(t) \quad (8a)$$

$$\Theta'_i = f'(t). \quad (8b)$$

In general, the gas flow-rate will be given as a function of time, while the air flow-rate $\dot{m}'$ is dictated by the following control† equations, which are derived in Appendix 1,

$$\dot{m}'(\Theta'_e - \Theta'_i)\bar{C}'_e = \dot{m}_b C'_b(\Theta_b - \Theta'_i) \quad \text{"series"} \quad (9)$$

$$\dot{m}'_1(\bar{C}'_e\Theta'_{e1} - \bar{C}'_{e2}\Theta'_{e2}) = \dot{m}_b(\bar{C}_b\Theta_b - \bar{C}_{e2}\Theta_{e2}) \quad \text{"staggered"}. \quad (10)$$

Moreover, the air exit temperature must satisfy the condition

$$\Theta'_e = \Theta_b \text{ at } t = t'_0 \text{ "series"} \quad (11a)$$

$$\Theta'_e = \Theta_b \text{ at } t = t'_0/2 \text{ "staggered"}. \quad (11b)$$

The heat-transfer coefficients h includes a radiation part h_r and a convective part h_c. The h_r is calculated with the methods described in the standard textbooks [11]. The graphs for the emissivities of CO_2 and H_2O and all correction factors have been included in the program in the form of tables. For the convective part h_c we have used Bohm's equation [12] given below, but any appropriate equation can be used.

$$h_c = (a_0 + a_1 u/D^{0.4})F_c F_R \quad \text{laminar} \quad (12a)$$

$$h_c = (a_2 u^{0.8}/D^{0.333})F_c F_R \quad \text{turbulent} \quad (12b)$$

where $a_0 = 5.4080$, $a_1 = 1.3665$, $a_2 = 3.381$

F_c, a temperature correction factor equal to $[(\Theta + 273)/1000]^{0.25}$ and F_R, roughness correction factors taken from [12].

The properties of the solid and gases and the heat-transfer coefficients are temperature dependent. They are evaluated in the time–space mean corresponding temperatures except for the specific heat of air which appears in the control equations (9, 10). It is given by the expression (13)

$$C' = b_1 + b_2\Theta' + b_3\Theta'^2 \quad (13)$$

with $b_1 = 987.1$, $b_2 = 0.2360$ and $b_3 = -0.3974 \times 10^{-4}$.

It is shown in Appendix 2, that with introduction of the dimensionless variables, ξ, ξ', η, η', the differential equations, the cyclic and boundary conditions, and the control equations become

$$\lambda \partial U/\partial \xi = F(\lambda)(V-U) \quad (14)$$

$$\partial V/\partial \eta = F(\lambda)(U-V) \quad (15)$$

$$\lambda' \partial U'/\partial \xi' = F(\lambda')(U'-V') \quad (16)$$

$$\partial V'/\partial \eta' = F(\lambda')(V'-U'). \quad (17)$$

For $0 \leqslant \xi \leqslant \Lambda$, $0 \leqslant \xi' \leqslant \Lambda'$, $0 \leqslant \eta \leqslant \Pi$, $0 \leqslant \eta' \leqslant \Pi'$

$$V(0, \xi) = V'[\Pi', (\Lambda' - \xi')] \quad (18a)$$

$$V(\Pi, \xi) = V'[0, (\Lambda' - \xi')] \quad (18b)$$

$$V_i = f(\eta) \quad (19a)$$

$$V_i = f'(\eta') \quad (19b)$$

$$\lambda' U'_e(B_1 + B_2 U'_e + B_3 U'^2_e) = U_b(B_1 + B_2 U_b + B_3 U_b^2) \quad (20)$$

$$\lambda(U'_{e1} - U'_{e2})[B_1 + B_2(U'_{e1} + U'_{e2}) + B_3(U'^2_{e1} + U'_{e1}U'_{e2} + U'^2_{e2})] = (U_b - U'_{e2})[B_1 + B_2(U_b + U_{e2}) + B_3(U'^2_{e2} + U'_{e2}U_b + U_b^2)] \quad (21)$$

where B_1, B_2, B_3 are constants and equation (20) refers to the series arrangement and (21) to the staggered.

The function $F(\lambda)$ (see Appendix 2) is

$$F(\lambda) = h^*/h^*_{\max} = (K_1 + K_2)/\{K_2 + 1/[F_1(\lambda) + K_3]\} \quad (22)$$

where K_1, K_2, K_3 constants and

$$F_1(\lambda) = h_c/h_{c\max} = (C_0 + C_1\lambda)/C_2 \quad \text{laminar} \quad (23a)$$

$$F_1(\lambda) = C_2\lambda^{0.8} \quad \text{turbulent} \quad (23b)$$

where C_0, C_1 and C_2 are constants. If for $\dot{m}_{\max}$ or $\dot{m}'_{\max}$ the flow is laminar, the denominator of (23a) should be replaced by $C_0 + C_1$. The critical value of λ, where the appropriate expression for laminar or turbulent should be used, is readily obtained by equating the numerators of (23a) and (23b).

We have also the auxiliary conditions

$$\left.\begin{array}{l} \lambda' = 1 \quad (24a) \\ U'_e = U_b \quad (24b) \end{array}\right\} \text{ at } \eta' = \Pi' \text{ "series"}$$

$$\lambda'_1 + \lambda'_2 = 1 \quad (25a)$$

$$\left.\begin{array}{l} \lambda'_1 = 1 \quad (25b) \\ U'_{e1} = U_b \quad (25c) \end{array}\right\} \text{ at } \eta' = \Pi'/2 \text{ "staggered"}.$$

3. FINITE DIFFERENCE REPRESENTATION

The partial differential equations (14)–(17) cannot be solved analytically, thus a numerical solution is employed, using the following finite difference scheme.

†The word control is used to indicate that there is a flow control device placed in the gas stream.

We integrate (14) from ξ to $\xi+\Delta\xi$ according to the trapezoidal rule [14],

$$\int_{\xi}^{\xi+\Delta\xi} \lambda(\partial U/\partial\xi)\,\mathrm{d}\xi = \int_{\xi}^{\xi+\Delta\xi} F(\lambda)(V-U)\,\mathrm{d}\xi \quad (26a)$$

$$\lambda[U(\xi+\Delta\xi)-U(\xi)] = (\Delta\xi/2)[V(\xi+\Delta\xi) + V(\xi)-U(\xi+\Delta\xi)-U(\xi)]+O(\Delta\xi)^3. \quad (26b)$$

Let the positions ξ, η be

$$\xi = n\Delta\xi \quad (27a)$$

$$\eta = j\Delta\eta \quad (27b)$$

where n,j are integers $0 \leqslant n \leqslant N$, $0 \leqslant j \leqslant J$.

Here

$$\Delta\xi = \Lambda/N \quad N \text{ total space node} \quad (28a)$$

$$\Delta\eta = \Pi/J \quad J \text{ total time steps.} \quad (28b)$$

Similar expressions are obtained in the same way for the partial differential equations (15)–(17) which in the usual notation are written,

$$\lambda^j(U^j_{n+1}-U^j_n) = (\Delta\xi/2)F^j(V^j_{n+1}+V^j_n-U^j_{n+1}-U^j_n) \quad (29)$$

$$V^{j+1}_n - V^j_n = (\Delta\eta/2)[F^{j+1}(U^{j+1}_n - V^{j+1}_n) - F^j(U^j_n - V^j_n)] \quad (30)$$

$$\lambda'^{j'}(U'^{j'}_{n'+1}-U'^{j'}_{n'}) = (\Delta\xi'/2)F'^{j'}(U'^{j'}_{n'+1}+U'^{j'}_{n'}-V'^{j'}_{n'+1}-V'^{j'}_{n'}) \quad (31)$$

$$V'^{j+1}_{n'} - V'^{j'}_{n'} = (\Delta\eta'/2)[F^{j'+1}(V'^{j'+1}_{n'} - U'^{j+1}_{n'}) + F^{j'}(V'^{j'}_{n'} - U'^{j'}_{n'})]. \quad (32)$$

The cyclic equilibrium conditions are,

$$V^0_n = V'^{j'}_{N-n'} \quad (33a)$$

$$V^J_n = V'^0_{N-n'} \quad (33b)$$

and the boundary conditions,

$$U^j_0 = F^j \quad (34a)$$

$$U'^{j'}_0 = f'^{j'} \quad (34b)$$

where $0 \leqslant n, n' \leqslant N$ space points (air and gas)

$$\left.\begin{array}{l} 0 \leqslant j \leqslant J \\ 0 \leqslant j' \leqslant J' \end{array}\right\} \text{ time steps.}$$

The control equations (20, 21) take now the form,†

Series:

$$\lambda^J U^j_N[B_1+B_2U^j_N+B_2(U^j_N)^2] = U_b(B_1+B_2U_b+B_3U_b^2) \quad (35a)$$

$$\lambda^J = 1. \quad (35b)$$

Staggered:

$$\lambda^j_1(U^j_{1N}-U^{j_2}_{2N})\{B_1+B_2(U^j_{1N}+U^{j_2}_{2N}) + B_3[(U^j_{1N})^2+U^j_{1N}U^{j_2}_{2N}+(U^{j_2}_{2N})^2]\} = (U_b-U^{j_2}_{2N})\{B_1+B_2(U_b+U^{j_2}_{2N}) + B_3[U_b^2+U_bU^{j_2}_{2N}+(U^{j_2}_{2N})^2]\} \quad (36a)$$

where

$$j_2 = j-J/2.$$

We also have the auxiliary conditions

$$\lambda^j_1+\lambda^{j_2}_2 = 1 \quad (36b)$$

$$\lambda^{J/2}_1 = 1. \quad (36c)$$

The heat transferred from the gas to the solid can be found by integrating equations (1) and (2).

$$Q_p = -cL\int_0^L\int_0^{t_0} \dot{m}\frac{\partial\theta}{\partial Z}\,\mathrm{d}z\,\mathrm{d}t = CV_s\int_0^L\int_0^{t_0}\frac{\partial T}{\partial t}\,\mathrm{d}t\,\mathrm{d}z \quad (37)$$

$$Q_p = c\langle\dot{m}[\Theta(0)-\Theta(L)]\rangle = CV_s[\bar{T}(t_0)-\bar{T}(0)] \quad (38)$$

Let $Q_0 = ct_0\dot{m}_{\max}\Delta\Theta$ then Q_p is equal to,

$$Q_p = Q_0\langle\lambda(U_i-U_e)\rangle = (\Lambda/\Pi)Q_0[\bar{V}(\Pi)-\bar{V}(0)]. \quad (39)$$

If the integration of (37) is performed according to the trapezoidal rule, then identical results are obtained from the finite-difference equations,

$$Q_p = Q_0\langle\lambda(U^j_0-U^j_N)\rangle = (\Lambda/\Pi)Q_0[\bar{V}_n(\Pi)-\bar{V}_n(0)]. \quad (40)$$

Similarly for the air side

$$Q'_p = Q'_0\langle\lambda'(U'^{j'}_N-U'^{j'}_0)\rangle = (\Lambda'/\Pi')Q'_0[\bar{V}'_n(0)-\bar{V}'_n(\Pi')]. \quad (41)$$

A criterion that the cyclic equilibrium has been reached is the quality of $Q_p = Q'_p$ and since $\bar{V}_n(\Pi) = \bar{V}'_n(0)$, $\bar{V}_n(0) = \bar{V}'_n(\Pi')$ we have the condition

$$\langle\lambda(U^j_0-U^j_N)\rangle = (Q'_0/Q_0)\langle\lambda'(U'^{j'}_0)\rangle \quad (42a)$$

where

$$Q'_0/Q_0 = (\Lambda/\Pi)/(\Lambda'/\Pi'). \quad (42b)$$

4. METHOD OF SOLUTION OF THE FINITE DIFFERENCE EQUATIONS

We start with an initial guess of $\langle\bar{\Theta}\rangle$, $\langle\bar{\Theta}'\rangle$, $\langle\bar{T}\rangle$, V^0_n. The space–time average temperatures will determine the quantities Π, Π', Λ, Λ', $\Delta\xi$, $\Delta\eta$ and the constants in the functions $F(\lambda)$ and $F(\lambda')$. Let us now assume that at time j all the V^j_n are known.

(a) Equation (29) is used to determine U^j_n since U^j_0 is known. From equation (30) we determine V^{j+1}_0. Then the two equations (29) (written for $j+1$) and (30) solved simultaneously will determine U^{j+1}_1, V^{j+1}_1. Continued in this fashion, we determine, step by step, all the U^{j+1}_n, V^{j+1}_n. When the gas period is completed, air flows through the regenerator in the counterflow direction.

(b) Here the program assumes a value for U_b. The equations (31), (32) are solved as in step (a), but $\lambda'^{j'}$ should now be compatible with the control equations. At the end of the air period, if the assumed and calculated blast temperatures are different, the calculations are repeated until the two values converge.

(c) Air and gas periods are alternately repeated for each stove until cyclic equilibrium is achieved, which is

†For simplicity the ′ from the equations (35)–(36c) has been deleted.

determined with the aid of equation (42).

At each cycle, the program corrects for the new calculated values of $\langle\Theta\rangle$, $\langle\Theta'\rangle$ and $\langle\bar{T}\rangle$. The program is designed to take into account the variation of the heat-transfer coefficient with temperature at each time step and at each node or at prescribed group of nodes.

For the series arrangement, the program monitors two stoves one being on air and one being on gas. In the staggered configurations, it monitors four stoves. The two stoves on air are 1/2 air period out of phase while the two stoves of gas are also 1/2 air period out of phase.

6. EXAMPLES

The method described previously of regenerator calculations can be used to obtain optimum designs of new stove systems and also to examine possible improvements of existing installations. To demonstrate its usefulness, we investigate here a four stove system operating under a "series" and "staggered" arrangement. As an example, we take the one cited in [7].

(A) *Physical dimensions of the regenerator*
- (i) Total heat-transfer area = $32940\,m^2$
- (ii) Volume of the solid = $572.8\,m^3$
- (iii) Cross section of flues = $12.7\,m^2$
- (iv) Height (3 zones) = 32.93 m
- (iva) zone 1 high duty = 20.73 m
- (ivb) zone 2 ufala = 6.10 m
- (ivc) zone 3 mullite = 6.10 m
- (v) Cylindrical flues diameter = 0.0508 m

(B) *Gas composition, pressure 1 atm*
- CO_2 0.1757 (by volume)
- H_2O 0.0608 (by volume)
- N_2O 0.75 (by volume)
- O_2 0.0185 (by volume)

(C) *Air composition, pressure 4 atm*
- Air 0.94 (by volume)
- H_2O 0.06 (by volume)

(D) *Roughness correction factor* 1.16

(E) *Emissivity of the solid* 0.8

(F) *Constant inlet temperatures*
- Gas inlet temperature 1440°C
- Air inlet temperature 93°C

(G) *Blast air conditions*

Required blast flow-rate	59 scms
Air period	5400 s
Desired blast temperature	1093°C (2000°F)
Reversal time	1800 s

(H) *Maximum gas exit temperature* $\leqslant$ 343°C

For any stove arrangement, the gas period t_0 is determined by the equation

$$t_0 = \frac{\text{No. of stoves on gas}}{\text{No. of stoves on air}} \times t'_0 - t_r. \qquad (43)$$

In order to have a meaningful comparison of the two systems, the hourly gas consumption of each stove should be kept constant. Thus, the gas flow-rate should satisfy the equation,

$$\frac{(\dot{m})\cdot t_0}{t_0 + t'_0 + t_r} = \text{constant}. \qquad (44)$$

In this example for the "series" arrangement $\dot{m}$ = 15.73 scms. Therefore, according to the above equations, we have the following data for the gas side,

(I) *"Series" arrangement*
- $\dot{m}$ = 15.73 scms
- t_0 = 14 400 s.

(J) *"Staggered" arrangement*
- $\dot{m}$ = 31.46 scms
- t_0 = 3600 s.

The truncation error for the blast temperature and the heat balance are taken equal to 0.0001 and the initial guess for $\langle\Theta\rangle$, $\langle\Theta'\rangle$, $\langle\bar{T}\rangle$, V_n^0 are 800°C, 670°C, 740°C and 0.7 respectively. The properties of the gas and air are calculated with the standard methods according to their composition and the solid properties are computed taking into account the length of each zone as described in [7]. There are four cases presented here,

Case I. "Staggered" with the heat-transfer coefficients calculated as functions of temperature at each node and each time step.

Case II. "Staggered" with the heat-transfer coefficients evaluated at $\langle\Theta\rangle$, $\langle\Theta'\rangle$, $\langle\bar{T}\rangle$.

Case III. "Series" with the heat-transfer coefficients evaluated at $\langle\Theta\rangle$, $\langle\Theta'\rangle$, $\langle\bar{T}\rangle$.

Case IV. "Series" with constant air flow-rate and equal to the average flow-rate of Case III.

All the calculations were performed in an IBM 360/75 digital computer with $N = 30$ nodes and $J = J' = 20$ time steps.

7. RESULTS AND DISCUSSION

In Table 1, the computed values of the blast temperature, maximum gas exit temperature, the average air and gas flow-rates, the thermodynamic efficiencies of air-side, gas-side, and regenerator are shown. In the last line, the required computer time for each case is also tabulated.

In Fig. 1, the variation of the dimensionless air flow-rate through the stove and the dimensionless air and gas temperatures for Case I is presented, while the same quantities for Case III are shown in Fig. 2. The distribution of the solid temperature over the height of the stove at the end of the cooling and heating periods for Cases I and III is shown in Fig. 3, and in Fig. 4 the variation of the solid temperature during the cycle for six different locations over the height of the stove for Cases I and III is plotted vs time. Finally, in Table 2 the time–space average of the heat-transfer coefficients solid conductance and the gas, solid and air temperatures are tabulated.

Inspection of Table 1, reveals that the Cases I and II give very close results while the computer time required is almost six times higher in Case I than in Case

Table 1.

Computed values	Case I	Case II	Case III	Case IV
Blast temperature, °C	1106	1096	1092	1097
Max. gas exit temp., °C	135	140	194	195
Average gas flow-rate, scms	29.50	29.50	52.46	52.46
Average air flow-rate, scms	31.46	31.46	15.73	15.73
Efficiency air side, η_t'	0.787	0.768	0.852	0.852
Efficiency gas side, η_t	0.977	0.974	0.954	0.953
Regenerator efficiency, η_r	0.872	0.859	0.900	0.900
Required computer time, min	9.183	1.564	0.358	0.132

Table 2.

Computed values		Case I	Case II	Case III	Case IV
h_{cmax}	$W/m^2\,°C$	37.9	38.4	39.6	36
$\langle h_c' \rangle$	$W/m^2\,°C$	27.9	27.4	36	36
$\langle h_c \rangle$	$W/m^2\,°C$	23.6	23.9	14.2	14.2
$\langle h_r \rangle$	$W/m^2\,°C$	8.1	7.6	10.8	10.8
$\langle h_c + h_r \rangle$	$W/m^2\,°C$	31.7	31.5	25.0	25.0
$3k/\delta\phi$	$W/m^2\,°C$	237.8	237.8	234.5	234.5
$\langle \Theta \rangle$	°C	561	550	662	664
$\langle \bar{T} \rangle$	°C	523	512	623	622
$\langle \Theta' \rangle$	°C	466	455	550	549

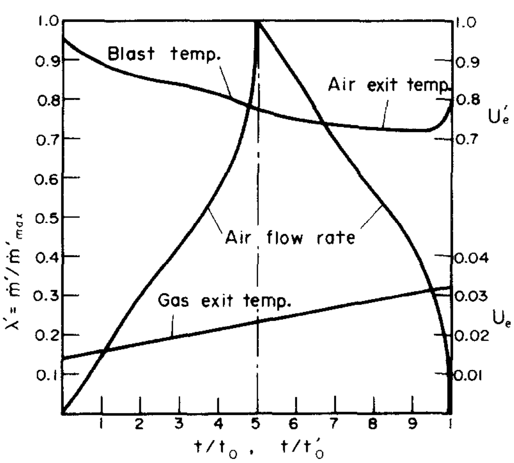

FIG. 1. Variation of dimensionless air exit temperature U_e', gas exit temperature U_e and air flow-rate λ' in the staggered system.

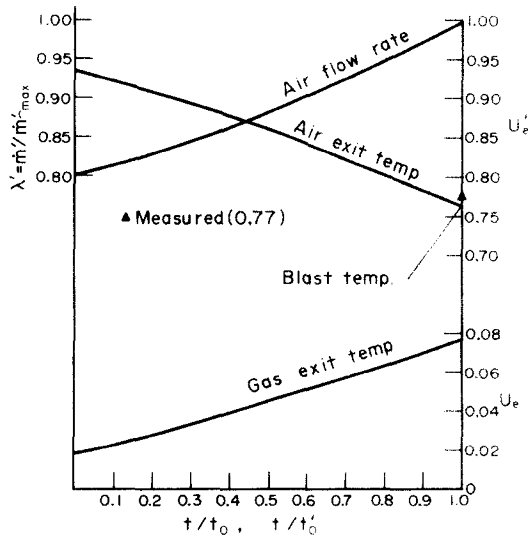

FIG. 2. Variation of dimensionless air exit temperature U_e', gas exit temperature U_e and air flow-rate λ' in the series four stove system.

II. The results for Case II indicate slightly lower blast temperature and a higher maximum gas exit temperature. Therefore, we conclude that one may perform the computations using heat-transfer coefficients which are calculated using time–space average temperatures. Thus, the results will be on the safe side (lower blast, higher max. gas exit temperatures) with a great economy in computer time.

We should mention here that the maximum gas exit temperature is an important feature of these regenerators, because it places structural limitations on the stoves and it should not exceed the values of 345°C. As it was expected, Cases III and IV give very close results. The influence of the stove arrangement is shown from the results of Cases II and III where similar type of calculations have been performed. Here the blast temperatures are almost the same in both cases (differ by 0.36%) but the maximum gas exit temperature in Case II is much lower (28% lower) than in Case III.

From Fig. 1 we can see that the blast temperature in a "staggered" arrangement is not the minimum air exit temperature through the stove as it is in a "series" arrangement (see Fig. 2) since the stove continues to operate for 1/2 period more. The gas exit temperature exhibits the same behavior in both arrangements. The air flow-rate is radically different in the two arrangements. In the "series" arrangement the variation is

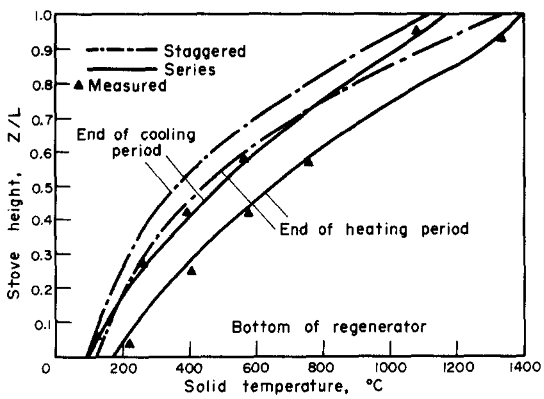

FIG. 3. Solid temperature distribution at the end of the periods.

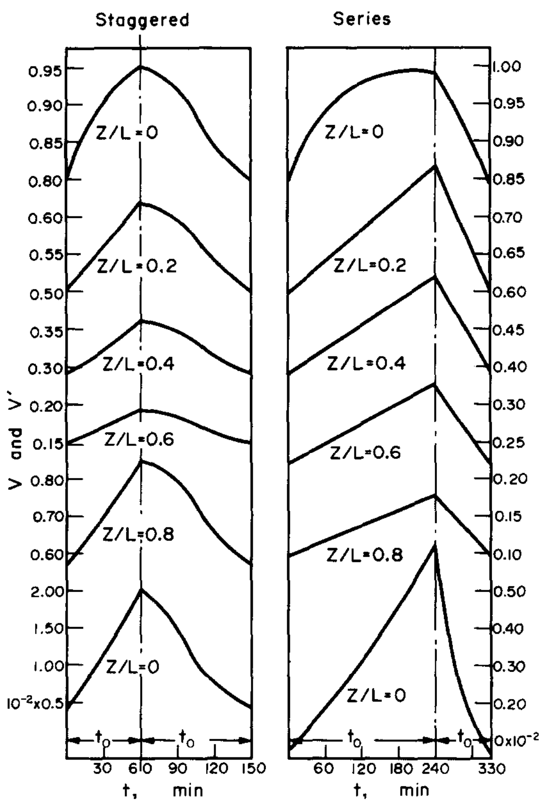

FIG. 4. Variation of solid temperature with time.

almost linear from 80% to 100% of the maximum flow-rate with a calculated average value of 89%. The solid temperature distribution shown in Fig. 3 are similar to those obtained by Willmott [15]. In Fig. 4 we observe that the variation of the solid temperature during the cycle is linear (except for the ends of the regenerator) in the "series" arrangement but not in the "staggered". One should remember that the linear variation of the solid temperature is the basic assumption for calculating the correction factor Φ (equation 31) which is now violated. However, the values of the wall conductance in the two Cases II and III differ only by 1.4%. Moreover, the wall conductance which is 237.8 W/m^2 °C (see Table 2) is 7.5 times higher than the fluid conductance of 31.5 W/m^2 °C. Thus, small variations in the correction factor Φ, would not have any appreciable effect on the results.

Let us now examine closely the performance of the two arrangements, Cases II and III. From the computed values of the maximum gas temperatures and their thermodynamic efficiencies, we can conclude that both systems are underutilized and they can probably produce higher blast temperatures by increasing the gas flow-rate until the limit of 345°C for the maximum gas exit temperature has been reached.

We have run several cases with different flow-rates and we have found that the "series" arrangement can allow an increase of 6% resulting in a blast temperature of 1124°C, while in the "staggered" arrangement, the gas flow-rate can be increased by 28% producing a blast temperature of 1296°C. Thus, in this four stove system, the "staggered" arrangement can produce by far better results (higher blast temperature) than the "series" arrangement. In Table 3, the computed values of the blast temperature, the maximum gas exit temperature, and the thermodynamic efficiencies of the stove for different gas flow-rates are tabulated. The limiting value of the 28% increase of the gas flow-rate was obtained by plotting the maximum gas exit temperature vs gas flow-rate and observe what flow-rate would produce approximately 345°C.

Table 3. "Staggered" arrangement

% Increase of flow-rate	Θ_b °C	Θ_e °C	η_t'	η_t	η_r
0	1096	190	0.768	0.974	0.859
10	1147	209	0.843	0.949	0.893
15	1174	240	0.868	0.035	0.900
20	1200	275	0.890	0.920	0.904
28	1296	343	0.921	0.889	0.905

8. CONCLUSION

A method is presented here for obtaining the solution of the non-linear regenerator problem where the air flow-rate is controlled directly from the air exit temperature. This control process expressed in mathematical terms has been simulated in a digital computer. The variable flow-rate introduces a heat-transfer process which is an explicit function of the time-varying flow-rate and an implicit function of the instantaneous solid and fluid temperatures. We have found that without great loss of accuracy, only the space–time average temperatures can be used for the approximate calculations, thus reducing the required computer time considerably. The known methods for taking into account the solid thermal resistance can be employed, even for systems which do not fulfill the basic assumption of a linear variation of the solid temperature. It is also shown how this method can be successfully utilized to investigate stove performance.

Acknowledgement—The authors gratefully acknowledge, the financial support of this work by the Bethlehem Steel Co.

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APPENDIX 1

Derivation of the control equations (9), (10), (20) and (21).

Consider the adiabatic mixing of two streams of air with flow-rates $\dot{m}_1$, $\dot{m}_2$ and specific enthalpies h_1 and h_2 respectively. The resulting stream will have a flow-rate $\dot{m}_3$ and specific enthalpy h_3 which will obey the following equations

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \tag{1.1}$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \tag{1.2}$$

where

$$h = \int_{\Theta_0}^{\Theta} c\, d\Theta \tag{1.3}$$

and Θ_0 a reference temperature.

From (1.1) and (1.2) we obtain

$$\dot{m}_1(h_1 - h_2) = \dot{m}_3(h_3 - h_2) \tag{1.4}$$

which by virtue of equation (1.3) becomes

$$\dot{m}_1[\bar{c}_1(\Theta_1 - \Theta_0) - \bar{c}_2(\Theta_2 - \Theta_0)] = \dot{m}_3[\bar{c}_1(\Theta_3 - \Theta_0) - \bar{c}_2(\Theta_2 - \Theta_0)] \tag{1.5}$$

where

$$\bar{c} = \frac{1}{\Theta - \Theta_0}\int_{\Theta_0}^{\Theta} c\, d\Theta. \tag{1.6}$$

Substituting the equation (1.5) of the values $\dot{m}_3 = \dot{m}_b$, $\dot{m}_1 = \dot{m}$, $\Theta_3 = \Theta_b$, $\Theta_1 = \Theta'_e$ and $\Theta_0 = \Theta'_i$ we obtain equation (9). Similarly for $\Theta_1 = \Theta'_{e1}$, $\Theta_2 = \Theta'_{e2}$ we obtain equation (10). The average volumetric specific heat is determined from (1.6) with the aid of the expression (13), thus

$$\bar{c}(\Theta - \Theta_0) = b_1(\Theta - \Theta_0) + \frac{b_2}{2}(\Theta^2 - \Theta_0^2) + \frac{b_3}{3}(\Theta^3 - \Theta_0^3). \tag{1.7}$$

For the most common case of constant air inlet temperature $\Theta_0 = \Theta'_i = \langle \theta_i \rangle U_i^0 = 0$ and $\Theta = U\Delta\Theta + \Theta_0$, equation (1.7) takes the form

$$\bar{c}(\Theta - \Theta_0) = \Delta\Theta b_1 U + \Delta\Theta \frac{b_2}{2} U(\Theta + \Theta_0) + \Delta\Theta \frac{b_3}{3} U(\Theta^2 + \Theta\,\Theta_0 + \Theta_0^2) \tag{1.8a}$$

or

$$\bar{c}(\Theta - \Theta_0) = \Delta\Theta U\left\{b_1 + \frac{b_2}{2}(U\Delta\Theta + 2\Theta_0) + \frac{b_3}{3}[(U\Delta\Theta)^2 + 3\Theta_0^2 + 3U\Theta_0\Delta\Theta]\right\}. \tag{1.8b}$$

Collecting the terms of equal powers of U in (1.8a) we obtain the expression

$$\bar{c}(\Theta - \Theta_0) = \Delta\Theta U[B_1 + B_2 U + B_3 U^2] \tag{1.9}$$

where

$$B_1 = b_1 + b_2\Theta_0 + b_3\Theta_0^2 \tag{1.10a}$$

$$B_2 = \frac{b_2}{2}\Delta\Theta + b_3\Theta_0\Delta\Theta \tag{1.10b}$$

$$B_3 = \frac{b_3}{3}\Delta\Theta^2. \tag{1.10c}$$

With the appropriate substitution of $U = U'_e$ etc. into the equation (1.9) we obtain the corresponding control equations (20) and (21).

If the specific heats can be considered constant (evaluated at their time average temperatures) then $B_1 = b_1$, $B_2 = B_3 = 0$ and the control equations take the simple form

$$\lambda' U'_e = U_b \quad \text{"series"} \tag{1.11}$$

$$\lambda'_1(U'_{e1} - U'_{e2}) = U_b - U'_{e2} \quad \text{"staggered"}. \tag{1.12}$$

APPENDIX 2

Derivation of the dimensionless differential equations (14) and (15).

The differential equations are.

$$h^*A(\Theta - T) = CV_s\frac{\partial T}{\partial t} \tag{2.1}$$

$$h^*A(T - \Theta) = \dot{m}cL\frac{\partial \theta}{\partial z}. \tag{2.2}$$

Let $h^*_{\max}$ be the corrected heat-transfer coefficient evaluated at the maximum flow-rate and at the time–space average temperatures $\langle \bar{\Theta} \rangle$, $\langle \bar{T} \rangle$, $\langle \bar{\Theta}' \rangle$ and $h_{c\max}$ the convective heat-transfer coefficient evaluated at the maximum flow-rate and at temperature $\langle \bar{\Theta} \rangle$. Introducing the variables

$$\xi = \frac{h^*_{\max}A_z}{\dot{m}_{\max}cL} \tag{2.3}$$

$$\eta = \frac{h^*_{\max}A_t}{CV_s} \tag{2.4}$$

and the function

$$F(\lambda)=\frac{h^*}{h^*_{max}} \qquad (2.5)$$

we obtain the equations (14) and (15).

The function $F(\lambda)$ is equal to,

$$F(\lambda)\frac{1/h^*_{max}}{1/h^*}=\frac{1/(h_{cmax}+h_r)+\Phi\delta/3k}{1/(h_c+h_r)+\Phi\delta/3k} \qquad (2.6)$$

or

$$F(\lambda)=\frac{1/(1+h_r/h_{cmax})+\Phi\delta h_{cmax}/3k}{1/(h_c/h_{cmax}+h_r/h_{cmax})+\Phi\delta h_{cmax}/3k}. \qquad (2.7)$$

When cyclic equilibrium has been established, h_{cmax}, h_r and $\Phi\delta/3k$ are constants and the function $F(\lambda)$ becomes

$$F(\lambda)=\frac{K_1+K_2}{K_2+1/[F_1(\lambda)+K_3]} \qquad (2.8)$$

where

$$K_1=\frac{1}{1+h_r/h_{cmax}} \qquad (2.9a)$$

$$K_2=\frac{\Phi\delta h_{cmax}}{3K} \qquad (2.9b)$$

$$K_3=\frac{h_r}{h_{cmax}} \qquad (2.9c)$$

and

$$F_1(\lambda)=\frac{h_c}{h_{max}}=\frac{C_0+C_1\lambda}{C_2} \quad \text{"laminar"} \qquad (2.10a)$$

$$F_1(\lambda)=\frac{h_c}{h_{cmax}}=C_2\lambda^{0.8} \quad \text{"turbulent"} \qquad (2.10b)$$

where according to the equations (12a), (12b) the constants C_0, C_1 and C_2 are

$$C_0=5.4080F_cF_R \qquad (2.11a)$$

$$C_1=\frac{1.3665F_cF_R}{D^{0.4}} \qquad (2.11b)$$

$$C_2=\frac{3.881F_cF_R}{D^{0.333}}. \qquad (2.11c)$$

We notice that the function $F(\lambda)$ depends explicitly on λ and implicitly on the temperatures $\langle\Theta\rangle$, $\langle\bar{T}\rangle$ and $\langle\Theta'\rangle$.

APPENDIX 3

The effect of longitudinal conduction in the Cowper stove regenerators is discussed by Willmott [16] and in a more general way by Bahnke and Howard [17].

According to [17] this effect is expressed in terms of a dimensionless conduction parameter γ defined as:

$$\gamma=\frac{kA_s}{\dot{m}cL}=\frac{k\{V/L-A_f\}}{\dot{m}cL} \qquad (3.1)$$

where A_s = the total solid area available for longitudinal conduction and A_f = the cross section area of the flues. It is shown that this effect for $\gamma<0.1$ and $\Lambda>10$ is directly proportional to γ. The regenerators discussed in this paper have values of $\gamma \ll 1$ and $\Lambda>10$. In the example given here we have,

$$A_s \doteq V/L-A_f=572.8\,m^2/32.93\,m-12.7\,m^2=4.69\,m^2.$$

An average $k=1.34\,W/m^2\,K$
$\dot{m}=315\,m^3/s$
$c=1140\,J/m^3\,K$.

Substituting the above values into (3.1) we obtain $\gamma=5.3\times10^{-7}$. The value of Λ in this example for all cases is greater than 15. Therefore the effect of longitudinal conduction is negligible.

MODELE DE CALCUL DES REGENERATEURS THERMIQUES AVEC DES DEBITS MASSIQUES VARIABLES

Résumé—Les échangeurs régénérateurs qui fonctionnent à température de sortie constante pour le fluide froid sont équipés avec un mécanisme de commande qui assure soit le contournement du régénérateur par une certaine quantité de fluide entrant et le mélange direct avec le fluide sortant, soit le contrôle du mélange de deux écoulements de sortie qui sont en déphasage moitié avec la période froide, de façon à maintenir constante la température. Dans les deux cas, le débit variable du fluide froid est commandé par sa température de sortie. Cet article décrit la solution des équations différentielles qui gouvernent ce processus thermique non linéaire. La dépendance des propriétés du solide et du fluide vis-à-vis de la température et le coefficient de transfert thermique sont pris en compte. Cette méthode peut être utilisée pour faciliter la conception de nouveaux régénérateurs ou pour vérifier les performances d'installations existantes. La solution est générale et applicable à n'importe quel type de régénérateur.

RECHENMODELL FÜR REGENERATOREN MIT VARIABLEN MASSENDURCHSÄTZEN

Zusammenfassung—Regenerative Wärmetauscher, die konstante Austrittstemperaturen auf der kalten Seite benötigen, sind mit Reglern ausgestattet, die entweder eine bestimmte Menge des einströmenden Fluids unter Umgehung des Regenerators direkt mit dem ausströmenden Fluid mischen, oder sie regeln bei zwei Regeneratoren die Mischung der beiden Austrittsströme. Die beiden Regeneratoren sind dabei um die Hälfte der kalten Periode phasenverschoben, um die konstante Temperatur zu erhalten. In beiden Fällen wird der unterschiedliche Massendurchsatz auf der kalten Seite mit der Austrittstemperatur geregelt. Diese Arbeit beschreibt die Lösung der Differentialgleichungen, welche diesen nicht-linearen Wärmeübergang bestimmen. Die Temperaturabhängigkeit des festen und flüssigen Zustandes und der Wärmeübergangskoeffizienten werden berücksichtigt. Diese Methode kann beim Entwurf von neuen Regeneratoren nützlich sein oder bei vorhandenen Regeneratoren die Wirksamkeit verbessern. Die Lösung ist allgemein und für jeden Regeneratortyp anwendbar.

РАСЧЕТНАЯ МОДЕЛЬ ДЛЯ ТЕПЛОВЫХ РЕГЕНЕРАТОРОВ С ПЕРЕМЕННОЙ СКОРОСТЬЮ ТЕЧЕНИЯ ЖИДКОСТИ

Аннотация — Регенеративные теплообменники, в которых охлажденная жидкость должна иметь постоянную температуру на выходе, снабжены для этой цели контрольным устройством, позволяющим или отводить определенное количество поступающей жидкости за пределы регенератора и затем смешивать ее непосредственно с выходящим потоком жидкости, или регулировать смешение двух потоков жидкости, исходящих из двух регенераторов, работа которых сдвинута по фазе на 1/2 холодного режима. В любом из двух случаев переменная скорость течения охлажденной жидкости контролируется ее температурой на выходе. В статье приводится решение дифференциальных уравнений, описывающих данный нелинейный процесс переноса тепла. В решении учитывается температурная зависимость между теплофизическими свойствами каркаса регенератора и жидкости, а также коэффициент теплообмена. Предлагаемый метод можно использовать для расчета новых регенераторов или улучшения рабочих характеристик существующих установок. Решение является общим и может быть применено к любому типу регенераторов.